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Vacuum energy reduction through destabilization: a unification of quintessence and a dynamical approach?

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Abstract

Although quintessence cosmologies seem to explain the amount of cosmological constant today, the required conditions are severe. For example, an extremely slowly varying and light scalar field that rolls toward the vanishing vacuum energy state is needed, which must satisfy the required initial conditions so that the energy density in matter is comparable to the energy density stored in quintessence today. Our first question is whether it is possible to explain why quintessence “knows” that the Universe is going toward the vanishing energy state. We show from a new perspective that the conventional $O(H)$ corrections may reproduce the required potential. Our second question is whether it is possible to obtain plausible amount of the potential energy density today without tuning initial conditions. Since our resultant potentials mimic quintessence, one may solve the coincidence problem using the conventional techniques for solving the coincidence problem in quintessence cosmology. However, in our scenario we have another option, which is qualitatively different from the one that have been discussed in quintessence cosmology. Besides quintessence cosmology, we show that there is a relation between a dynamical approach, which suggests that we can solve the coincidence problem in a dynamical approach. As far as we know, this is the first attempt to discuss about the unified picture and the important relations between these distinctive scenarios.

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1 Introduction

The existence of a cosmological constant that is driving the acceleration of the present Universe is perhaps the most striking result that has been obtained in modern cosmological observations. Related to the existence of a cosmological constant, there is a serious problem; we must explain the value of the energy density stored in the cosmological constant today, which has to be comparable to the critical density. Although many possibilities have been discussed by many authors², it may still be fair to say that there is no successful explanation as to why the cosmological constant appears at the present size. Among many proposals, quintessence has been proposed as the candidate of the varying cosmological constant, which is a dynamical, slowly evolving component with negative pressure. A key problem with the quintessence proposal would be explaining why the energy density stored in the quintessence and the matter energy densities are comparable today. There are two major aspects to the above coincidence problem; one is that the initial condition in the early Universe has to be tuned so that the energy densities become comparable today, and the other is that the quintessence energy density is very tiny compared to typical scale of particle physics. The former condition may be softened by introducing a form of quintessence called tracker fields, and the latter may be forced by direct measurements.

Although quintessence cosmology seems attractive, it still needs further explanations so as to why quintessence is an extremely slowly varying and light scalar field that rolls toward the vanishing energy state. One may find a kind of solution in the string theory or brane models, however it is still very difficult especially to explain why the quintessence is rolling down the potential toward the vacuum with vanishingly small energy density. In this paper, we will consider the above questions trying to find explanations why the quintessence has its peculiar properties. We show from a new perspective that the conventional $O(H)$ corrections may explain both (1) the smallness of the potential energy density and (2) the coincidence between the potential energy density and the energy density in matter.³ Starting with simple setups, we will “obtain” potentials that mimic

²See ref.[1] and references therein.

³Here we use the word “ $O(H)$ corrections” to mean the effective terms that appear with H^n , where H is the Hubble constant.

quintessence.

Moreover, we will show how one can generalize the conventional quintessence cosmology so that it may accommodate itself to the “coincidence” in the present Universe.

Our mechanism can be used to solve the coincidence problem in a dynamical approach.

2 Eta-problem and $O(H)$ corrections

Let us briefly review $O(H)$ corrections that appear in scalar potential. For example, there are many theories where one has effective mass with $|\Delta m^2| = O(H^2)$. If the field ϕ non-minimally couples to gravity, it may acquire a correction to the mass squared $|\Delta m^2| \simeq \xi R$, where R is the curvature scalar. For the popular choice of conformal coupling $\xi = 1/6$, one obtains $|\Delta m^2| = 2H^2$. A similar situation appears in cosmological models with supergravity, where scalar fields acquire correction $|\Delta m^2| \simeq H^2$. A similar correction appears also in brane inflationary models, where light scalar fields may acquire correction to the mass squared due to the mechanism that stabilizes the internal space. Since these corrections are not favorable in obtaining successful inflationary expansion in the early Universe, the appearance of the above terms is called “eta-problem”.

Before we discuss about our mechanisms, we will explain some details about the $O(H)$ corrections. To be precise, the conventional “ $O(H)$ corrections” do not always mean the corrections of $O(H)$. For example;

- In conventional supergravity, “ $O(H)$ corrections” are corrections from F -terms, not from the Hubble constant. Of course, they may coincide if the F -term dominates the Hubble constant, however we must be careful about this statement. This idea is used in D -term inflation models, since the “ $O(H)$ corrections” do not appear (or much smaller than the Hubble constant) if the dominant part of the inflationary potential is given by D -terms.
- There may be corrections from the thermal plasma, which is given by

$$|\Delta m^2| \propto \frac{T^4}{M_p^2}. \quad (2.1)$$

This term mimics “ $O(H)$ corrections” if the thermal plasma dominates the Universe.

- In KKLT models, flat directions may obtain $O(H)$ masses due to the mechanism that stabilizes the extra dimensions.

Although the magnitude of these corrections may be of the same order, these effects are qualitatively different from each other and may have different coefficients.

From the above arguments, it will be natural to introduce a new definition of the so-called “ $O(H)$ corrections”, introducing a new variable H_s , which is given by

$$H_s^2 \equiv (A_r \rho_r + A_m \rho_m + A_v \rho_v) / M_p^2, \quad (2.2)$$

where $A_{r,m,v}$ are the model-dependent coefficients, as we have mentioned above. Here ρ_r , ρ_m and ρ_v are the energy density in radiation, matter, and potential, respectively. In this paper, we will consider “ $O(H_s)$ corrections” instead of the usual $O(H)$ corrections. In our argument, the coefficients $A_{r,m,v}$ may take either sign, since each correction may appear independently.⁴

In addition to the effective mass terms that we have discussed above, one may include higher corrections

$$\Delta m^2(H_s, \phi) = H_s^2 f_1 \left(\frac{\phi}{M_*} \right) + H_s^4 f_2 \left(\frac{\phi}{M_*} \right) + \dots, \quad (2.3)$$

which may be given by

$$\begin{aligned} \Delta m^2(H_s, \phi) = & H_s^2 \left(B_{1,0} + B_{1,1} \frac{\phi^2}{M_*^2} + \dots + \frac{B_{1,i_*} \phi^{2i_*}}{M_*^{2i_*}} \right) \\ & + H_s^4 \left(\frac{B_{2,0}}{M_*^2} + B_{2,1} \frac{\phi^2}{M_*^4} + \dots + \frac{B_{2,i_*} \phi^{2i_*}}{M_*^{2(i_*+1)}} \right) + \dots, \end{aligned} \quad (2.4)$$

where B ’s are model-dependent coefficients, and i_* , j_* are cut-off parameters. We introduced M_* , which denotes the cut-off scale of the effective theory. One may take $M_* \simeq M_p$ for simplicity.

3 Destabilized potential

Before we go into details of the mechanism, it will be better to discuss more about the situations that we will consider in this paper. The important component in our mechanism

⁴Be sure that we are discussing about possibilities. It is still very hard to calculate the coefficients.

is the scalar field ϕ whose potential is destabilized by $O(H)$ corrections. As a result, the potential energy density is reduced when the field ϕ rolls down the potential.

First, let us consider a scalar field ϕ , which is flat at the tree level. The potential for the field ϕ is given by the simplest form,

$$V(\phi)_{tree} = V_0. \quad (3.1)$$

It is not surprising if the above potential is destabilized when the field ϕ obtains $O(H_s)$ corrections, which is given by

$$\Delta V(\phi) = -\frac{1}{2}H_s^2\phi^2, \quad (3.2)$$

where $A_v > 0$ is assumed. As we are considering H_s instead of H , the correction does depend on the coefficients $A_{r,m,v}$. If the above correction appears with the negative sign, as we have shown above in eq.(3.2), the potential $V(\phi)$ is destabilized, and then, the field ϕ starts to roll down the potential.

Let us consider the evolution of the field ϕ in a region $0 < \phi < M_*$. As the effective mass term (3.2) destabilizes the potential, the field ϕ rolls away to reduce the total potential energy density. In this region, the potential $V(\phi)$ is given by

$$\begin{aligned} V(\phi) &= V_0 + \int_0^\phi \left[\frac{d}{d\phi} \Delta V \right] d\phi \\ &\simeq V_0 - \frac{1}{2}A_v V(\phi) M_p^{-2} \phi^2, \end{aligned} \quad (3.3)$$

where we have assumed that the vacuum energy dominates H_s ;

$$H_s^2 \simeq A_v V(\phi) / M_p^2. \quad (3.4)$$

From the above expression, we obtained the form of the potential, which is given by

$$V(\phi) \simeq \frac{V_0}{1 + \frac{A_v \phi^2}{2M_p^2}}. \quad (3.5)$$

If the effective mass of the field ϕ is small, the approximate form of the above potential for $A_v < 1$ and $\phi \ll M_p$ is given by

$$V(\phi) \simeq V_0 \left(1 - \frac{A_v \phi^2}{2M_p^2} \right) \simeq V_0 - \frac{1}{2}H_s|_{\phi=0} \phi^2. \quad (3.6)$$

On the other hand, if the potential is very steep (i.e. $A_v \gg 1$), the approximate form of the above potential in the region $M_* > \phi > M_p \sqrt{2/A_v}$ is given by

$$V(\phi) \simeq \frac{2M_p^2 V_0}{A_v} \phi^{-2}, \quad (3.7)$$

if the region exists.

As we will discuss later in this section, the above form of ΔV may be altered in the outer region ($\phi > M_*$) where higher terms may start to dominate the potential.

As we are considering a scalar field ϕ that rolls down the potential with the negative mass term (3.2), the roll of the field ϕ may satisfy the condition for fast-roll inflation. To be more precise, if the initial value of ϕ is small and the coefficient satisfies the condition $A_v \leq \frac{3}{4}$, there will be a significant amount of fast-roll inflation[2, 3] that lasts until $\phi = M_*$, where higher terms may become significant. We found that the total number of e-foldings corresponding to fast-roll inflation is given by

$$N = \frac{1}{F} \ln \left(\frac{M_*}{\phi_0} \right), \quad (3.8)$$

where $\phi_0^2 \geq H^2 \sim V_0/M_p^2$, and the coefficient F is given by

$$F = \frac{3}{2} \left[1 - \sqrt{1 - \frac{4}{3}|A_v|} \right] < \frac{3}{2}. \quad (3.9)$$

Unlike the conventional scenario of fast-roll inflation, the effective mass is not a constant in our model. The effective mass decreases with time, since we are considering the mass term that comes from the $O(H_s)$ correction.

If one is considering a theory with many flat directions, the above destabilization may occur for the large number of $n \gg 1$ scalar fields. Then, assuming (for simplicity) spherical symmetry for n scalar fields, the factor of $n \gg 1$ appears in (3.3) as

$$V(\phi) = V_0 + n \int_0^\phi \left[\frac{d}{d\phi} \Delta V \right] d\phi. \quad (3.10)$$

We obtained the approximate form of the potential, which is given by

$$V(\phi) \simeq \frac{V_0}{1 + \frac{nA_v\phi^2}{2M_p^2}}. \quad (3.11)$$

In this case, even if the potential satisfies the fast-roll condition for each field, the reduction of the vacuum energy occurs much faster than the single-field potential.

What is important here is the qualitative property that the field ϕ keeps on rolling as far as the effective mass term appears with the negative sign, while the driving force disappears when $H_s \simeq 0$ at a distance. Of course, the vacuum energy cannot vanish because the field becomes extremely light before it crosses $H_s = 0$. In our model, the peculiar field that “knows” the zero-energy vacuum is not required, but it is obtained from the conventional effective potential. This property is distinguishable. In fact, any field can accommodate itself to the vanishing potential energy if only it satisfies the above conditions.

Let us discuss more about the destabilized potential $V(\phi)$. The above correction (3.2) is plausible within the region $0 < \phi < M_*$, where M_* is a cut-off scale. Above the cut-off scale M_* , higher corrections will be important. Then, for example in a region $M_* < \phi < \phi_1$, where the value of ϕ_1 is model-dependent, we can assume that the potential is dominated by a term coming from the higher corrections

$$\Delta V(\phi) = -a_{j,k,l} M_*^{-j} \phi^k H_s^l. \quad (3.12)$$

First we consider a case with $j, k \gg 1$ and $l = 2$. The potential is given by

$$\begin{aligned} V(\phi) &= V|_{\phi=M_*} + \int_{M_*}^{\phi} \left[\frac{d}{d\phi} \Delta V \right] d\phi \\ &= V|_{\phi=M_*} + \left[-a_{k-2,k,2} M_*^{-(k-2)} \phi^k H_s^2 + a_{k-2,k,2} M_*^{-(k-2)} M_*^k H_s^2 \right] \\ &= \hat{V}|_{\phi=M_*} - a_{k-2,k,2} M_*^{-(k-2)} \phi^k H_s^2, \end{aligned} \quad (3.13)$$

where the definition of \hat{V} is

$$\hat{V} \equiv V(\phi) + a_{j,k,l} M_*^{-j} \phi^k H_s^l. \quad (3.14)$$

Assuming that the vacuum energy dominates H_s , we obtained

$$V(\phi) \simeq \frac{\hat{V}|_{\phi=M_*}}{1 + a_{k-2,k,2} A_v M_*^{-(k-2)} \phi^k M_p^{-2}}. \quad (3.15)$$

At first, near $\phi \sim M_*$, this potential may appear to be a steep function of ϕ . On the other hand, the approximate form of the potential in the region $\phi_1 > \phi > \phi_c > M_*$ is given by

$$V(\phi) \simeq \Lambda_* \phi^{-k}, \quad (3.16)$$

where ϕ_c and Λ_* are defined as

$$\phi_c \equiv (M_*^{k-2} M_p^2 / a_{k-2,k,2} A_v)^{1/k} \quad (3.17)$$

and

$$\Lambda_* \equiv \frac{\hat{V}|_{\phi=M_*} M_*^{k-2} M_p^2}{a_{k-2,k,2} A_v}, \quad (3.18)$$

respectively. Note that the above potential mimics quintessence.

Seeing the above results, one may think that our mechanism considered here is the mechanism that explains the origin of the quintessential potential, revealing especially the reason why quintessence is extremely light and is rolling down to the vanishing energy state. This speculation is true, but we have more to discuss about.

Before discussing “more” about our model, let us further consider a case where ΔV in a region (for example) $M_* < \phi_* < \phi$ is dominated by the correction with $l = 4$, where the potential is given by

$$\begin{aligned} V(\phi) &= V|_{\phi=\phi_*} + \int_{\phi_*}^{\phi} \left[\frac{d}{d\phi} \Delta V \right] d\phi \\ &\simeq \hat{V}|_{\phi=\phi_*} - a_{k,k,4} M_*^{-k} \phi^k H_s^4. \end{aligned} \quad (3.19)$$

Assuming that the vacuum energy dominates H_s , we obtained

$$V(\phi) \simeq \frac{P}{2} \left(-1 + \sqrt{1 + \frac{4\hat{V}|_{\phi=\phi_*}}{P\phi^{-k}}} \right) \phi^{-k}, \quad (3.20)$$

where P is given by

$$P \equiv \frac{M_*^k M_p^4}{a_{k,k,4} A_v^2}. \quad (3.21)$$

Here the solution with the negative sign is neglected because one cannot reach at the negative solution. Again, we obtained a potential that may explain the peculiar properties of quintessence.

4 Coincidence problem?

Let us explain another new idea, which may explain why the order of the magnitude of the potential energy density is comparable to the energy density in matter today. To be more precise, we will examine if one can obtain $\rho_v \sim \rho_m$ in the above scenario, without using the ideas that have been considered in quintessential cosmology. What we will consider here is a case where the coefficient A_m appears with the negative sign, which

is the opposite to the sign of A_v . For now, we will assume $|A_r| \ll 1$ and disregard the energy density in radiation ρ_r .⁵ Then, the explicit form of H_s^2 is given by

$$H_s^2 = A_v \rho_v - A_m \rho_m, \quad (4.1)$$

where A_m is redefined to be positive. In our scenario, the effective potential for the field ϕ is destabilized if $H_s^2 > 0$ (i.e. $A_v \rho_v > A_m \rho_m$), where the reduction of the potential energy proceeds through the mechanism that we have stated above. On the other hand, if the energy density in matter becomes large and H_s^2 approaches to $H_s^2 \simeq 0$, the reduction of the vacuum energy is suspended.

If there is phase transition, the potential energy density may (partially) be converted into the energy density in matter. If the resultant energy density in matter is sufficiently large, the field ϕ will “rolls back” the potential.

The reduction mechanism is active as far as the energy density in matter satisfies the condition

$$\rho_v > \frac{A_m}{A_v} \rho_m, \quad (4.2)$$

while the field ϕ becomes extremely light when

$$\rho_v \simeq \frac{A_m}{A_v} \rho_m, \quad (4.3)$$

and then, it may start to dominate the energy density of the Universe in the later epoch. The vacuum energy ρ_v is positive and decreasing with time, and its later evolution is determined by the equation of motion for the field ϕ .

One may think that our mechanism considered here gives a “reason” to the initial conditions that has been considered in quintessence cosmology. This speculation is true, but we have another “more” to discuss about.

Now, before discussing about another “more”, let us consider energy density in radiation. What happens if the Universe is dominated by radiation? Assume that the coefficient A_r is very small in a period of the Universe, then the potential energy density at that time can be much smaller than the energy density in radiation (ρ_r) by a factor of A_r/A_v . Then, while the energy density in radiation decreases with time, the potential energy density decreases keeping the above factor.

⁵It is straightforward to include ρ_r in the above arguments.

It is easy to apply the above arguments to a realistic evolutionary scenario of the Universe. The evolution of the Universe after reheating will be determined by the energy density in radiation, when the coefficient A_r is very small ($|A_r| \ll 1$). In a later epoch, when the energy density in matter becomes significant, the potential energy density is bounded below by the ratio A_m/A_v . More later evolution of the field ϕ is determined by the equation of motion, which looks like quintessence. In our model, the evolution of the field ϕ is not simple but very sensitive to the changes in the Universe. Of course, the coefficients may also vary with time, especially at phase transitions when there are significant changes in the Universe. We will postpone the model-dependent analyses to our forthcoming paper.

5 A dynamical approach

There has been many proposals that might solve the cosmological constant problem dynamically. Here we consider a model named “a dynamical approach” that has been advocated by Mukohyama and Randall[5]. The novel feature is that a scalar field φ exists which has non-standard kinetic terms whose coefficient diverges at zero curvature. The divergent coefficient of the kinetic term means that the lowest energy state is never achieved, even if the minimum of the potential seems to have negative energy. Instead, the cosmological constant automatically stalls at or near zero.

To explain the basic idea of a dynamical approach, let us consider the following action

$$I = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} + \alpha R^2 + \beta \frac{K^q}{\kappa^4 f(R)^{2q-1}} - V(\varphi) \right], \quad (5.1)$$

where K is a function given by $K = \kappa^4 \partial_\mu \varphi \partial^\mu \varphi$. Here it is assumed that $f(R)$ behaves as

$$f(R) \sim (\kappa^2 R^2)^m. \quad (5.2)$$

If $q > 1/2$, the factor $\sim 1/f(R)^{2q-1}$ that appears in front of the kinetic term becomes very large as R approaches 0. As a result, the field φ becomes extremely slow-roll and cannot reach at the vacuum with vanishing energy density.

We found this interesting idea when we were considering about the relation between our scenario and quintessence cosmology, and thought that there must also be a relation

between our scenario and a dynamical approach. If so, we may find a unified picture of these (seemingly different) approaches, which will provide us with a powerful tool to settle the problems that arise in each model, through exchanging views among them. In fact, in the above model of a dynamical approach, it is possible to rescale the field φ as $\varphi \rightarrow R^{m(2q-1)}\varphi$ to absorb the factor in front of the kinetic term into the redefinition of φ , and then, due to the redefinition of the field φ , the factor is moved into the redefined potential $V(\varphi)$, which will be given by the form that is similar to eq.(3.12). Here we can use the relation between R and H , which is given by

$$R = -6(2H^2 + \dot{H}), \quad (5.3)$$

where \dot{H} is negligible if φ is a extremely slow-roll field. Obviously, the above relation and the redefinition of the field φ suggests that it is possible to reproduce our scenario from the action that has been used in a dynamical approach.

Now it is very easy to find the solution to the coincidence problem, which was thought to be a serious problem in a dynamical approach. In fact, in a dynamical approach the problem was serious because they have neglected other components (i.e, elements other than the scalar curvature R) that may appear in f . Of course, there is no reason to believe that other components (for example, kinetic terms and potentials) that appeared in the original action cannot appear in f . Considering more generic forms of f , we will define a new function $f(R_s)$, where a new variable R_s is used instead of the scalar curvature R . R_s is not a scalar curvature. It is defined so that one can use the relation

$$R_s \simeq H_s^2. \quad (5.4)$$

The new variable R_s and the new function $f(R_s)$ modifies the scenario of a dynamical approach. The most important discrepancy between the original scenario and our modified one is that the factor $\sim 1/f(R_s)^{2q-1}$ that appears in front of the kinetic term becomes very large as R_s (not R) approaches 0. As a result, the field φ becomes extremely slow-roll and cannot reach at the vacuum with $R_s = 0$. By analogy with our scenario, it is very clear that the coincidence problem in a dynamical approach is not the problem in a dynamical approach itself, but the problem in the definition of $f(R)$ and $V(\varphi)$. To be precise, the original extension of the kinetic term is not generic in a sense that it does not

contain all the allowed components in the theory. Considering more generic extension of the kinetic term, one can obtain the solution to the coincidence problem in a dynamical approach, using the mechanisms that we have discussed above in the previous section.

6 Conclusions and discussions

Both inflationary and quintessence cosmologies require scalar fields which roll slowly over cosmological time scales. The condition seems quite severe in quintessence cosmologies, where one needs to introduce an extremely slowly varying and light scalar field that must roll toward the vanishing vacuum energy state. In this paper, we started with a question whether it is possible to “explain” the above properties of quintessence cosmologies.

First, we showed from a new perspective that the conventional $O(H)$ corrections can be used to solve both the cosmological constant problem and the coincidence problem. Besides the fact that the resultant potentials mimic quintessence, we have a new idea to solve the coincidence problem, which is qualitatively different from the usual techniques that has been used in quintessence cosmologies.

In this paper we considered $O(H)$ corrections to solve the problems, while they are in some cases notorious for generating unwanted masses for the inflaton field. For brane inflationary models, the inflationary scenario that has been discussed in ref.[4] may circumvent the serious eta-problem.⁶ It is also notable that the mechanism presented in ref.[4] may induce a tilted spectrum that is determined by the inner structure of the extra dimensions. We are not going back to historical developments, however it is easy to understand that the idea of extra dimensions is very attractive. The idea of large extra dimensions is important, since it may solve or weaken the hierarchy problem. To be more precise, in models with large extra dimensions, fields in the standard model(SM) are localized on a wall-like structure, while the graviton propagates in the bulk. The discrepancy of the volume factor between gauge fields and gravitational fields may explain the large

⁶In ref.[4] we mainly considered a model in which fluctuations are generated along the equipotential surface, which is flat by definition. Application of elliptic inflation to brane inflationary models is very interesting. We also discussed in ref.[4] that $O(H)$ corrections lifting the inflationary direction may not become a serious problem if there is a symmetry enhancement at the tip of the inflationary throat.

hierarchy between gravity and gauge interactions[6]. An important progress in brane cosmology is made in ref.[7], where a variant of D-term inflation is found in the paradigm of brane cosmology[8, 9]. In any case, it is an important challenge to find signatures of branes in cosmological observations. Historically, this challenge has been made in a new paradigm of brane defects[10, 11].⁷ Inflation models with low fundamental scale are important if extra dimensions are large[13, 14]. Obtaining the baryon asymmetry of the Universe in such low-scale models is very difficult, but there are models that may solve this problem [15, 16, 17]. In these models, cosmic defects play distinguishable roles and may leave their signatures[18, 19, 20, 21, 22, 23, 24, 25]. The curvatons[26, 27, 28] may also be significant[29, 30], and is expected to leave their signatures on CMB.

In this paper, we considered $O(H)$ corrections that destabilize the potential for the field ϕ , and explained how one can obtain tiny cosmological constant. Our model can explain why quintessence rolls slowly down to the vanishing energy state. Another distinguishable property of our scenario is that we can explain the reason why the order of the magnitude of the cosmological constant is comparable to the energy density in matter today, without tuning the initial conditions. Our result suggests that there will be no serious problem in obtaining desired cosmological constant if the model has the properties that we have discussed in this paper. Unfortunately, it is still very hard to calculate the coefficients of the higher corrections that are very important to examine the model-dependent evolution. We think further study on this issue is needed, which may reveal the particular reason why we are living in the Universe with extremely tiny but substantial amount of cosmological constant.

Finally, we will make some comment about a principle that is required to solve the cosmological constant problem. The action that describes gravitational interactions seem to have a shape that is recursively constructed by the elements that appears in the usual formulation, which may be referred to as a fractal-like formulation. For example, in a dynamical approach, the coefficient in front of a kinetic term may depend on any element that appears in the original action. As we have discussed in this paper, the idea is quite important because it may be used to reproduce the peculiar form of the quintessential potential and also to explain why we are living with vanishingly small but substantial

⁷Supergravity provides us with a natural mechanism for removing cosmological domain walls[12].

amount of cosmological constant. However, if one considers only the scalar curvature R , one will find that the coincidence is a severe problem. The coincidence problem may be solved if one includes “other elements” in the function, which is of course more generic than considering only the scalar curvature R . Any “coefficient” that appeared in the original action may be extended to a generic function that depends on any element that appeared in the original action. As a result, the action is recursively constructed. Extension of the original gravitational action has been discussed by many authors. As far as we know, this is the first attempt to explain successfully the cosmological constant problem and the coincidence problem, reproducing the famous quintessence cosmology and a dynamical approach from the simple setups.

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